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SUPER (*a; d*)-EDGE ANTIMAGIC TOTAL LABELING OF CONNECTED SUNFLOWERS GRAPH

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Abstract

A graph$G$ of order p and size q is called an(a,d)-edge-antimagic if there exist a bijection $f : V(G) \cup E(G) \to \{1,2,\dots,p+q\}$such that the edge-weights$w(uv) = f(u) + f(v) + f(uv),\ uv \in E(G)$, form an arithmetic sequence with first term term a and common difference$d$. Such a graph$G$is called super if the smallest possible appear on the vertex labels. In this paper, we study super(a,d)-edge-antimagic total labeling connected properties of Sunflowers. The result shows that Sunflowers Graph admit a super edge antimagic total labeling for $d \in 0,1,2$.

Keywords: *super* (a,d)*-edge-antimagic total labeling, Sunflowers*.

I. Introduction

Topics of interest in graph theory is a graph labeling problem. One of a kind type of graph labeling is super (a,d)-edge-antimagic total labeling or super edge antimagic total labeling (SEATL). In this study, we investigate super (a,d)-edge-antimagic total labeling on Sunflower Connective Graph. Graph Sunflowers are one graph isomorphic developed from a *Cycle* graph by adding multiple *paths* skip a point outside *Cycle* forming like a Sunflower. The purpose of this study was to determine whether Graf Sunflower has a super (a,d)-edge-antimagic total labeling.

Graph Sunflower denoted by $\beta_{n,\ \frac{n-1}{2}}$for $n \geq 3$. Sunflowers in a graph that can be developed is the crowning piece to $\frac{n-1}{2}$. Graph Sunflowers have a set of *vertices*, $V\beta_{n,\ \frac{n-1}{2}} = \left\{x_i, y_j; 1 \leq i \leq n, 1 \leq j \leq \frac{n-1}{2}\right\}$ and edge set, $\beta_{n,\ \frac{n-1}{2}} = \left\{y_{(j)}, x_{(2j-1,k)}; 1 \leq j \leq \frac{n-1}{2};\right\} \cup \left\{y_{(j)}, x_{(2j+1)}; 1 \leq j \leq \frac{n-1}{2}\right\} \cup \left\{x_{(n)}, x_{(1)}\right\} \cup \left\{x_{(i)}, x_{(i+1)}; 1 \leq i \leq n-1\right\}$. So $|v| = \frac{3n-1}{2}$ and $|e| = 2n-1$

II. Methods

The method used in this research is axiomatic deductive, ie by lowering the axioms or theorems that have been there, then applied in the labeling of total super (a, d)–vertex antimagic on graph Sunflower either singly or combined with each other detachment. In this study, the first to be determined is the value of the difference (d) in a graph Sunflowers, then the value of d is applied in the labeling of total super (a, d)-edgeantimagic on graph Sunflower. If there is a total super labeling (a, d)- edgeantimagic, it will be defined how the labeling pattern of total super (a, d)- edgeantimagic on graph Sunflowers by using pattern detection methods (pattern recognition) to determine the general pattern.

Super (a, d)-edge-antimagic total labeling is a to by one mapping f from $V(G) \cup E(G)$to integers $\{1,2,3, \dots, p + q\}$such that the set weights sides $w(uv) = f(u) + f(v) + f(uv)$on all sides G is $\{a, a + d, a + 2d, \dots, a + (q - 1)d\}$for $a > 0$ and $d \geq 0$ both integers. A (a, d)-edge-antimagic total labeling called super (a, d)-edge-antimagic total labeling if $f(V) = \{1,2,3, \dots, p\}danf(E) = \{p + 1, p + 2, .., p + q\}$ To search for an upper limit value of the difference d labeling of total super (a, d)-edge-antimagic can be determined by lemma 1 This follows as (in Dafik: 2007).

Lemma 1*If a graph$(p, q)$is the labeling of total super(a, d)-edge antimagic thend* $\leq \frac{2p+q-5}{q-1}$

Proof. Suppose the graph (p, q) has a total super labeling of (a, d)-edge antimagic for $f(V) = \{1,2,3, \dots, p\}$and $\{f(E) = p + 1, p + 2, \dots, p + q\}$and mapping $: V(G) \cup E(G) \rightarrow \{1,2, \dots, p + q\}$. Minimum possible value of the weight from the smallest side is to add two smallest vertexes label (1 and 2) with one side of the smallest label $(p + 1)$, in order to obtain:

$1 + (p + 1) + 2 = p + 4$. If the set value of the side of a graph $\{a, a + d, a + 2d, \dots, a + (q - 1)d\}$in which a is of the smallest value side, it can be written $p + 4 \leq a$.

On the other hand, the maximum possible value of the weight of the largest side is to add two biggest vertexes labels ($(p - 1)$and p) with the biggest label side of (p + q), in order to obtain:

$(p-1) + (p+q) + p = 3p + q - 1$. The SEATL weight of nature which states that $a + (q-1)d$ is the largest tribe, it is obtained:

$$\leftrightarrow a + (q-1)d \leq 3p + q - 1$$
$$\leftrightarrow (p+4) + (q-1)d \leq 3p + q - 1$$
$$\leftrightarrow d \leq \frac{3p+q-1-(p+4)}{q-1}$$
$$\leftrightarrow d \leq \frac{2p+q-5}{q-1}$$

Lemma 2*A (p,q)-graph $G$is super antimagic if and only if the bijective function $f: V(G) \rightarrow \{1,2,\dots,p\}$ so that the set $S = \{f(u) + f(v): uv \in E(G)\}$consists of integers q respectively. In such case, f extends to a super edge-magic labeling of G with magic constant $a = p + q + s$, where $s = min(S)$ and $S = \{a-(p+1), a-(p+2), \dots, a-(p+q)\}$.*

In our terminology, the previous lemma states that a(p,q)-graph G is super $(a,0)$edge-antimagic total if and only if there exists an $(a-p-q,1)$-edge-antimagic vertex labeling.

This research was conducted at the Graph Single Sunflower. The steps of research are: (1) calculate the number of sides p and q on a number of points $\beta_n, \quad \frac{n-1}{2}$; (2) determining the upper limit of the difference d; (3) determine the label EAVL *(Edge Antimagic Vertex Labelling)* of $\beta_n, \quad \frac{n-1}{2}$; (4) if the label EAVL form a pattern, then proceed with determining EAVL bijective functions; (5) label side based on value EAVL and determine the function of a single broom bijektive on graph for each applied for possible; (6) determining SEATL bijective function (Super-Edge Antimagic Total Labeling) $\beta_n, \quad \frac{n-1}{2}$; (7) determining the bijective function EAVL *diskonektif* using techniques colouring graph with chromatic number = 3.

III. Result and Discussion

Super (a,d)-edge-antimagic total labeling on Graph Sunflower $\beta_{n,\frac{n-1}{2}}$

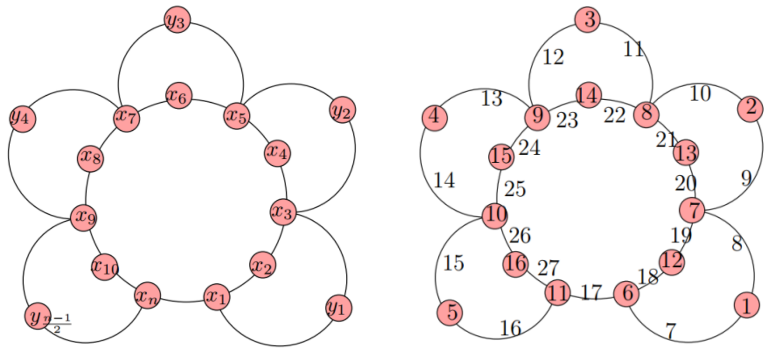

Figure 1. EAV Sunflowers Graph

If $\beta_{n,\frac{n-1}{2}}$ has a super total labeling of (a,d)-edgeantimagic for $p = \frac{3n-1}{2}$ and $q = 2n-1$, based on the lemma 1 upper limit value of d is $d \leq 2$ or $d \in \{0,1,2\}$. Lemma 1 **is lemma relating to the labeling point$(a,1)$-edgeAntimagic on $2n-1$**

Theorem 1*If $n \geq 3$ then the graph Sunflowers$\beta_{n,\frac{n-1}{2}}$ has a super$(\frac{n+3}{2},1)$-edge antimagic vertex labeling*

Proof. Label point Graph Sunflower $\beta_{n,\frac{n-1}{2}}$ with α_1 bijective function defined as a labeling $f_1 : V\left(\beta_{n,\left\lceil\frac{n-1}{2}\right\rceil}\right) \to \left\{1,2,\dots,\frac{3n+2}{2}\right\}$then labeling f_1 can be written as follows:

$$f_1(y_j) = j, \text{for} 1 \leq j \leq \frac{n-1}{2}$$

$$f_1(x_i) = \frac{n+i}{2}, \text{for} 1 \leq i \leq n, i \in odd$$

$$f_1(x_i) = \frac{2n+i}{2}, \text{for} 1 \leq i \leq n, i \in even$$

If w_{f_1} is defined as the value of hand labeling f_1 point, then the formulation of w_{f_1}, as follows:

$$w_{f_1}(y_j x_{2j-1}) = \frac{4j+n-1}{2}, \text{for} 1 \leq j \leq \frac{n-1}{2}$$

$$w_{f_1}(y_j x_{2j+1}) = \frac{4j+n+1}{2}, \text{for} 1 \leq j \leq \frac{n-1}{2}$$

$$w_{f_1}(x_n x_1) = \frac{3n+1}{2}$$

$$w_{f_1}(x_i x_{i+1}) \quad = \frac{3n+2i+1}{2}, \text{ for } 1 \leq i \leq n \leq \frac{n-1}{2}$$

The formulation of the form the set $\bigcup_{c=7}^{27} w_{f_1}^{c} = \left\{7,8,9,\ldots,\frac{5n-1}{2}\right\}$. Thus, it can be concluded that $f_1$is a labeling point $\left(\frac{n+3}{2},1\right)$.

Theorem 2 *There is a Super* $(4n,0)$*-edge-antimagic total labeling on graph Sunflower Single* $\beta_{n,\quad\frac{n-1}{2}}$*for* $n \geq 3$.

Proof. Use labelingf_1 point to label the point Graph Sunflower $\beta_{n,\quad\frac{n-1}{2}}$, then define the label side of $f_2 : E_{\left(\beta_{n,\quad\frac{n-1}{2}}\right)} \rightarrow \{4n,4n,..,4n\}$, so that the label side of f_2 then w_{f_2} can be formulated as follows:

$$w_{f_2(x_i x_{i+1})}^{1} = \frac{5n-2i-1}{2} + \frac{3n+2i-1}{2} = \frac{8n}{2}$$
$$w_{f_2(x_n x_1)}^{2} \quad = \frac{5n-1}{2} + \frac{3n-1}{2} = \frac{8n}{2}$$
$$w_{f_2(y_j x_{2j+1})}^{3} = \frac{7n-4j-1}{2} + \frac{4j+n+1}{2} = \frac{8n}{2}$$
$$w_{f_2(y_j x_{2j+1})}^{4} = \frac{7n-4j+1}{2} + \frac{4j+n-1}{2} = \frac{8n}{2}$$

If w_{f_2} is defined as the weight of the total Graph Sunflower labeling based on the sum value of the side with the label side, then w_{f_2} can be obtained by formulating a total val of the EAVL w_{f_1} and formula f_2 label side can be written as: $\bigcup_{c=17}^{37} w_{f_2}^{c} = \{4n,4n,\ldots,4n\}$. It can be concluded that the graph sunflower$\beta_{n,\quad\frac{n-1}{2}}$with $n \geq 3$, has a super (a,d)-edge-antimagic total labeling with $a = 4n$ and $d = 0$, in other words Graph Sunflower $\beta_{n,\quad\frac{n-1}{2}}$ has super $(4\mathrm{n},0)$-edge-antimagic total labeling.

Theorem 3*There is a super* $(2n+2;2)$*-edgeantimagic total labeling on graph Sunflower* $\beta_{n,\quad\frac{n-1}{2}}$ *for* $n \geq 3$

Proof. Label the vertex of Sunflower Graph $\beta_{n,\quad\frac{n-1}{2}}$ with $f_3(x_i) = f_1(x_i)$and $f_3(y_j) = f_1(y_j)$, define the label side f_3: $E_{\left(\beta_{n,\quad\frac{n-1}{2}}\right)} \rightarrow \{2n+2,2n+4,..,6n-2\}$, then the label side for labelingf_3 super $(a,2)$-edgeantimagictotal on Sunflower $\beta_{n,\quad\frac{n-1}{2}}$ can be formulated as follows:

$$f_3(y_j k^x{}_{2j-1}) = \frac{3n+4j-3}{2}, \text{for} 1 \le j \le \frac{n-1}{2}$$
$$f_3(y_j k^x{}_{2j+1}) = \frac{3n+4j-1}{2}, \text{for} 1 \le j \le \frac{n-1}{2}$$
$$f_3(x_n k^x{}_{1,k}) \ = \frac{5n-1}{2}$$
$$f_3(x_i k^x{}_{i+1,k}) \ = \frac{5n+2i-1}{2}, \text{for} 1 \le i \le n$$

If $w_{f_3}$is defined as the total value of the labeling side by labeling f_3 then w_{f_3} can be formulated as follows:

$$w^1_{f_3(y_j x_{2j-1})} = \frac{3n+4j-3}{2} + \frac{4j+n-1}{2} = \frac{4n+8j-4}{2}$$
$$w^2_{f_3(y_j x_{2j+1})} = \frac{3n+4j-1}{2} + \frac{4j+n+1}{2} = \frac{4n+8j}{2}$$
$$w^2_{f_3(x_n x_1)} \quad = \frac{5n-1}{2} + \frac{3n+1}{2} = \frac{8n}{2}$$
$$w^2_{f_3(x_i x_{i+1})} = \frac{5n+2i-1}{2} + \frac{3n+2i+1}{2} = \frac{8n+4i}{2}$$

The formulation forms the set $\bigcup_{c=17}^{37} w^c_{f_3} = \{2n+2, 2n+4, \dots, 6n-2\}$. It means Sunflower Graf $\beta_n, \quad \frac{n-1}{2}$ has a total labeling of super (a, d)-edgeantimagic with $a = 2n + 2$dan $d = 2$.

Theorem 4 *the labeling of total super* $\left(\frac{6n+2}{2}, 1\right)$*-edgeantimagic on graph Sunflower* $\beta_n, \quad \frac{n-1}{2}$*forn* ≥ 3.

Proof. Label vertex of Sunflowers Graph $\beta_n, \quad \frac{n-1}{2}$ with $f_4(x_i) = f_1(x_i)$and $f_4(y_j) = f_1(y_j)$, define the label side of $f_4 : E_{\left(\beta_n, \quad \frac{n-1}{2}\right)} \to \left\{\frac{6n+2}{2}, \frac{6n+4}{2}, \dots, \frac{10n-2}{2}\right\}$, the label on the side f_4Sunflowers Graph $\left(\beta_n, \quad \frac{n-1}{2}\right)$ can be formulated:

$$f_4(y_j x_{2j-1}) = \frac{5n-2j+1}{2}, \text{for} 1 \le j \le \frac{n-1}{2}$$
$$f_4(y_j x_{2j+1}) = \frac{7n-2j-1}{2}, \text{for} 1 \le j \le \frac{n-1}{2}$$
$$f_4(x_i x_{i+1}) \quad = \frac{4n-1}{2}, \text{for} 1 \le i \le n-1,\ i \in \text{even}$$
$$f_4(x_i x_{i+1}) \quad = \frac{6n-i-1}{2}, \text{for} 1 \le i \le n-2,\ i \in \text{odd}$$
$$f_4(x_n x_1) \quad\ = \frac{4n}{2}$$

If w_{f_4} is defined as the total value of the labeling side by labeling f_4 then w_{f_4}, it can be obtained by the following formula:

$$w^1_{f_4(y_j x_{2j-1})} = \frac{5n-2j+1}{2} + \frac{4j+n-1}{2} = \frac{6n+2j}{2} \text{for} 1 \le j \le \frac{n-1}{2}$$

$$w^1_{f_4(y_j x_{2j+1})} = \frac{7n-2j-1}{2} + \frac{4j+n+1}{2} = \frac{8n+2j}{2} \text{for} 1 \le j \le \frac{n-1}{2}$$

$$w^1_{f_4(x_i x_{i+1})} = \frac{4n-i}{2} + \frac{3n+2i+1}{2} = \frac{7n+i+1}{2} \text{for} i \in \text{even}$$

$$w^1_{f_4(x_i x_{i+1})} = \frac{6n-i-1}{2} + \frac{3n+2i+1}{2} = \frac{9n+i}{2} \text{for} i \in \text{odd}$$

$$w^1_{f_4(x_n x_1)} = \frac{4n}{2} + \frac{3n+1}{2} = \frac{7n+1}{2}$$

The above formulation can be written in the set $\bigcup_{c=17}^{37} w^t_{f_4} = \left\{\frac{6n+2}{2}, \frac{6n+4}{2}, \dots, \frac{10n-2}{2}\right\}$. It means that Sunflower Graph $\beta_{n,\ \frac{n-1}{2}}$ has a total labeling of super (a, d)-edgeantimagic with $a = \frac{6n+2}{2}$ and $d = 1$ or Graph Sunflowers $\beta_{n,\ \frac{n-1}{2}}$ have Super $\left(\frac{6n+2}{2}, 1\right)$.

Joint Construction Technique of Disconnected Sunflowers Graph. The method used in finding the super total labeling of (a, d)-edge antimagic on the combined Sunflower Graph s$\beta_{n,\ \frac{n-1}{2}}$ by finding the chromatic number = 3 that gives the number of colors to a minimum point, wherein each vertex which neighbors are not the same color. Labelling EAV *diskonektif* researchers developed a new entry using graph coloring.

Lemma 3 Suppose $\boldsymbol{\Upsilon}$ is a set of numbers in sequence $\Upsilon = \left\{a, a + \frac{k+2}{2}; a + 1; a + \frac{k+4}{2}; a + 2; a + \frac{k+6}{2}, \dots, a + \frac{2k}{2}; a + \frac{k}{2}\right\}$where k is even, then there is a permutation Ω and Φ of the members of the set Y such that Ω + Y, Φ + Y and Ω + Φ is also a set of numbers in sequence.

Proof: Ω + Y is a set of numbers in sequence. Suppose $\boldsymbol{\Upsilon}$ is the set of numbers in sequence $\Upsilon = \left\{a, a + \frac{k+2}{2}; a + 1; a + \frac{k+4}{2}; a + 2; a + \frac{k+6}{2}, \dots, a + \frac{2k}{2}; a + \frac{k}{2}\right\}$and k is even, then defined $\Upsilon = \{v_t{}^{\Upsilon} | 1 \le t \le k\}$:

Where $v_t{}^{\Upsilon} = \begin{cases} a + \frac{t-1}{2}, \text{for } 1 \le t \le k+1, \;\; t \in \text{odd} \\ a + \frac{k+t}{2}, \text{for } 1 \le t \le k, \;\; t \in \text{even} \end{cases}$

Similarly defined permutation $\Omega = \{w_t{}^{\Omega} | 1 \le t \le k\}$ of $\boldsymbol{\Upsilon}$ members as follows:

$$w_t{}^{\Omega} = \begin{cases} a + \dfrac{t-2}{2}, \text{for } 1 \le t \le k, \;\; t \in \text{even} \\ a + \dfrac{k+t-1}{2}, \text{for } 1 \le t \le k+1, \;\; t \in \text{odd} \end{cases}$$

With direct evidence we get $v_t{}^{\Upsilon}$ and Ω + Y expressed in the set $w_t{}^{\Omega}$ then obtained:

$$\Omega + \Upsilon = \{w_t{}^{\Omega} + v_t{}^{\Upsilon} | 1 \le t \le k\}$$

$$= \left\{\left(a + \tfrac{t-1}{2}\right) + \left(a + \tfrac{k+t-1}{2}\right), 1 \le t \le k+1, \;\; t \in \text{odd}\right\} \cup$$

$$\left\{\left(a + \tfrac{k+t}{2}\right) + \left(a + \tfrac{t-2}{2}\right), 1 \le t \le k, \;\; t \in \text{even}\right\}$$

$$\Omega + \Upsilon = \left\{2a + \tfrac{k}{2}, 2a + \tfrac{k+2}{2}, 2a + \tfrac{k+4}{2}, 2a + \tfrac{k+6}{2}, \ldots, 2a + \tfrac{3k-2}{2}, 2a + \tfrac{3k}{2}\right\}$$

It is evident there is a permutation Ω of the members of the set $\boldsymbol{\Upsilon}$ so that Ω + $\boldsymbol{\Upsilon}$ is also a set of numbers in sequence.

Φ + $\boldsymbol{\Upsilon}$ is a set of numbers in sequence. Furthermore, the value defined permutation $\Phi = \{w_t{}^{\Phi} | 1 \le t \le k+1\}$ of $\boldsymbol{\Upsilon}$ members as follows: $w_t{}^{\Phi} = a + k - t + 1$ for$1 \le t \le k+1$, with direct evidence we get $v_t{}^{\Upsilon}$and + $\boldsymbol{\Upsilon}$expressed in the set $w_t{}^{\Phi}$maka obtained:

$$\Phi + \Upsilon = \{w_t{}^{\Omega} + v_t{}^{\Upsilon} | 1 \le t \le k\}$$

$$= \left\{\left(a + \tfrac{t-1}{2}\right) + (a + k - t + 1), 1 \le t \le k+1, \;\; t \in \text{odd}\right\} \cup$$

$$\left\{\left(a + \tfrac{k+t}{2}\right) + (a + k - t + 1), 1 \le t \le k, \;\; t \in \text{even}\right\}$$

$$\Phi + \Upsilon = \left\{2a + k, 2a + \tfrac{3k}{2}, 2a + k - 1, 2a + \tfrac{3k-2}{2}, \ldots, 2a + \tfrac{2k+2}{2}, 2a + \tfrac{k}{2}\right\}$$

It is evident there is a permutation Φ of the members of the set $\boldsymbol{\Upsilon}$so that Φ + $\boldsymbol{\Upsilon}$ is also a set of numbers in sequence.

Ω + Φ is a set of numbers in sequence. Suppose Φ is a set of numbers in sequence $= \{a + k, a + k - 1, a + k - 2, a + k - 3, \ldots, a + 1, a\}$ and k even $\boldsymbol{\Phi} = \{v_t{}^{\Phi} | v_t = a + k - t + 1, 1 \le t \le k+1\}$with direct evidence we get $v_t{}^{\Phi}$and $\boldsymbol{\Omega}$ + $\boldsymbol{\Phi}$ expressed in the set $w_t{}^{\Omega}$then obtained:

$$\Omega + \Phi = \{v_t{}^{\Phi} + w_t{}^{\Omega} | 1 \le t \le k\}$$

$$= \left\{(a + k - t + 1) + \left(a + \tfrac{k+t-1}{2}\right), 1 \le t \le k+1, \;\; t \in \text{odd}\right\} \cup$$

$$\left\{(a+k-t+1)+\left(a+\frac{t-2}{2}\right)+, 1 \le t \le k,\ \ t \in \text{even}\right\}$$

$$\Omega + \Phi = \left\{2a+\frac{3k}{2}, 2a+k-1, 2a+\frac{3k-2}{2}, 2a+k—2, \dots, 2a+\frac{k}{2}, 2a+\frac{2k}{2},\right\}$$

It is evident there is a permutation Ω of the members of the set Φ that $\boldsymbol{\Omega}$ + $\boldsymbol{\Phi}$ is also a set of numbers in sequence.

The combined construction techniques independently in graph Sunflowers can be immediately lowered to EAVL *diskonektif*bijective function that is summing function $\boldsymbol{\Upsilon}$,, with EAVL $\beta_n,\ \ \frac{n-1}{2}$sole namely:

$$f(u) = f(\Upsilon, \Omega, \Phi) + [f(v) - 1].(k+1), \text{untuk } 1 \le t \le k$$

IV. Conclusion

Based on the results of the previous discussion, it can be concluded that:

1. There is a super$\left(\frac{n+3}{2}, 1\right)$-edge antimagicvertex labelingSunflower Graphfor $n \ge 3$it is evidenced that Theorem 1.
2. There is a super $(4n, 0)$-edge antimagicTotal Labeling of Connected Sunflowers Graph single $\beta_n,\ \ \frac{n-1}{2}$for $n \ge 3$it is evidenced that Theorem 2.
3. There is a super $(2n+2{,}2)$-edge antimagictotal labeling of Connected Sunflowers Graph$\beta_n,\ \ \frac{n-1}{2}$for $n \ge 3$it is evidenced that Theorem 3.
4. There is a super $\left(\frac{6n+2}{2}, 1\right)$-edgeantimagictotal labeling of Connected Sunflowers Graph$\beta_n,\ \ \frac{n-1}{2}$for $n \ge 3$it is evidenced that Theorem 4.
5. Suppose $\boldsymbol{\Upsilon}$ is a set of numbers in sequence $\Upsilon = \left\{a, a+\frac{k+2}{2}; a+1; a+\frac{k+4}{2}; a+2; a+\frac{k+6}{2}, \dots, a+\frac{2k}{2}; a+\frac{k}{2}\right\}$where k is even, then there is a permutation Ω and Φ of the members of the set $\boldsymbol{\Upsilon}$ such that $\boldsymbol{\Omega}$ +$\boldsymbol{\Upsilon}$, $\boldsymbol{\Phi}$ + $\boldsymbol{\Upsilon}$ and $\boldsymbol{\Omega}$ + $\boldsymbol{\Phi}$ is also a set of numbers in sequence, it is evidenced that Lemma 3.

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